Physics 34 Equation Sheet

FRONT

Maxwell's Equations

$\Phi_B = \int \vec{B}\, d\vec{A}$

$\Phi_E = \int \vec{E}\, d\vec{A}$

Gauss' $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

Ampere's $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \overbrace{\left(\mu_0 \varepsilon_0 \frac{d}{dt}\Phi_E\right)}^{\text{displacement current}}$

- between capacitors
- Maxwell's Law of induction

Faraday's $\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B$

no monopoles $\oint \vec{B}\cdot d\vec{A} = 0$

$\mu_0 = 4\pi \times 10^{-7}\ \frac{T\cdot m}{A}$

$\varepsilon_0 = 8.85\times 10^{-12}\ \frac{C^2}{N m^2}$

$k = \frac{1}{4\pi\varepsilon_0} = 9\times 10^9\ \frac{Nm^2}{C}$

Maxwell's Wave Equations

$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$

$\frac{\partial^2 B}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B}{\partial^2 t}$ $\quad E_0 = cB_0$

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $\quad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ ↳ Bounded

$c = 3\times 10^8\ m/s$

Distribution Functions

Boltzmann's ↳ $P(s) = \frac{1}{Z} e^{-\frac{E(s)}{k_B T}}$

T (temp, K°)

$k_B = 8.617\times 10^{-5}\ \frac{eV}{K}$

Z = partition function
s = state

avg X $\langle X\rangle = \sum_s X(s)\cdot P(s)$

$P(s) = \frac{N(s)}{N_{tot}}$

$P = \sigma A T^4$ → Blackbody Power

$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2}$

Stat-Mech

$\langle X\rangle = \sum_s X(s)\, P(s)$

$\langle X\rangle = \frac{\int_0^\infty x P(x) dx}{\int P(x) dx}$

$P(s) = \frac{N(s)}{N_{tot}}$

$n_i = g_i F_B(s)$

$\langle E\rangle = \sum_s \frac{E(s) N(s)}{N_{tot}}$

$\langle E\rangle = \frac{\int_0^\infty E n(E) dE}{(N_{tot})}$

$n(E) = g(E) F_B$

$\frac{N}{vol} = \int n(E) dE$

Relativity

$\Delta t = \gamma \cdot \Delta t'$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$\beta = \frac{v}{c}$

$L = \frac{L'}{\gamma} = L'\sqrt{1-\frac{v^2}{c^2}}$

Waves

Wave # $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$ $\quad v = f\lambda$

$v_{rms} = \sqrt{\langle v^2\rangle}$ $\quad p = \hbar k$ $\quad \omega = \frac{2\pi}{T}$

$v_{rms} = \sqrt{\frac{3k_B T}{m}}$

$k = \frac{n\pi}{L}$

?↳ $k = n\frac{\pi}{L}$!

$E_p = (n)hf = pc$

$h = 6.63\times 10^{-34}\ J s$

↳ $= 4.13\times 10^{-15}\ eV s$

$hc = 1240\ eV\cdot nm$

$B = B_0 \cos(k(x-vt))$ (n)
$E \quad E_0$

$I_0 = cB_0$

Quantum Physics

$E = -\frac{ke^2}{2r_n}$

$r_n = \frac{n^2\hbar^2}{m k e^2 Z}$ $\quad k = 9\times 10^9\ \frac{N m^2}{C^2}$

$\hbar = \frac{h}{2\pi} = 2.715\times 10^{-30}$ eV·s

$r_n = \frac{a_0 n^2}{Z}$ Bohr

$a_0 = 0.529\ Å$
↖ ground state hydrogen

Atomic Physics

Rydberg's

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$R = 1.09\times 10^7\ m^{-1}$

$E = \frac{-13.6\ eV}{n^2}(Z)^2$ ← E_R
↖ proton #

$E = \frac{-13.6\ eV}{n^2}(Z-1)^2$
↖ screening

$2d\sin\theta = n\lambda$
↖ Bragg's (crystal)

$L = n\hbar$ ← bohr momentum

Lorentz transformations

$S \to S'$

$x' = \gamma(x - vt)$

$t' = \gamma\left(t - x\frac{\beta}{c}\right)$

$v_p' = \frac{v_p - v}{1 - v_p\frac{v}{c^2}}$

$y = y'$
$z = z'$

$S' \to S$

$x = \gamma(x' + vt')$

$t = \gamma\left(t' + x'\frac{\beta}{c}\right)$

$v_p = \frac{v_p' + v}{1 + v_p'\frac{v}{c^2}}$

$\vec{p}_\gamma = \gamma m_p \vec{v}_p$
m_r

$E_T = \gamma mc^2$

$K = (\gamma - 1)mc^2$

$(E)^2 = (pc)^2 + (mc^2)^2$

$E = pc$

$E' = E_0\sqrt{\frac{1\pm\beta}{1\mp\beta}}$ Doppler ← num ⊕ towards

Compton's ↓
$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta_{photon})$

$\lambda = \frac{h}{mc}$ not - actual

$100\ Å = 10\ nm$ actual

$1\ Å = 10^{-10}\ m = 10^{-1}\ nm$

$1\ eV = 1.6\times 10^{-19}\ J$

$ke^2 = 1.4\ eV\cdot m$

Quantum $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2$ BACK

Schrödinger's Equation

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi_{(x,t)} + U(x)\Psi_{(x,t)} = i\hbar\frac{\partial}{\partial t}\Psi_{(x,t)}}$$

$\lambda = \frac{h}{p}$ $E = pc$

$K_n = \frac{2\pi(n)}{\lambda}$

Heisenburg's uncertainty

$\Delta x \Delta p \geq \frac{\hbar}{2}$ → $\sigma_x \sigma_p \geq \frac{\hbar}{2}$

$\Delta t \Delta E \geq \frac{\hbar}{2}$ → $\sigma_t \sigma_E \geq \frac{\hbar}{2}$

$\cancel{\Delta x \Delta k \geq \frac{1}{2}}$

$\sigma_j = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$

$$|\Psi_{(x,t)}|^2 = \Psi^*_{(x,t)} \cdot \Psi_{(x,t)} = \rho$$ ← Probability density

$i \to -i$ complex conjugate

$c = a + ib$

$c^* = a - ib$

$e^{i\theta} = \cos\theta + i\sin\theta$

$|\Psi_{(x,t)}|^2 = |\Psi_{(x)}|^2$

$|\Psi_{(x,t)}|^2 = P_{\text{of particle in } dx}$

Born's law

$A^2\int |\Psi_{(x,t)}|^2 dx = 1$

all space

normalization constant

$\sum_{n=1} |C_n|^2 = 1$

$\hat{KE} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}[\ ]$ $\hat{P} = \frac{\hbar}{i}\frac{\partial}{\partial x}[\ ]$ $\hat{H}[\ ] = (KE + U_{(x)})\Psi_{(x)}$

$\hat{P}[\Psi] = p\Psi$

$\hat{H}[\Psi(x)] = E\Psi(x)$

$E = \hbar\omega$ $E = \omega\hbar$

general wave function → $\Psi_{(x,t)} = \sum \Psi_{(x,t)} e^{-i\frac{E_n}{\hbar}t}$

$\phi(t) = e^{-i\omega t} = e^{-\frac{iEt}{\hbar}}$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi_{(x)} + U\Psi = E\Psi}$$

TISE

Wave Function constraints

I. $\lim_{x\to\pm\infty} \Psi_{(x,t)} = 0$

II. $\lim_{x\to\pm\infty} \frac{\partial}{\partial x}\Psi_{(x,t)} = \text{bound}$

$\int |\Psi_{(x,t)}|^2 dx = \int \rho\, dx$ → Probability $x \to x + dx$

$\Psi_{(x,t)} = \Psi_{(x)}\phi_{(t)}$

superposition $\Psi(x, t=0) = \sum_{n=1}^{\infty} C_n \Psi_n(x)$

$\Psi_{(x,t)} = \sum_{n=1}^{\infty} C_n \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)e^{-\frac{iE_n t}{\hbar}}$

$C_n = \sqrt{\frac{2}{a}}\int_0^a \sin\left(\frac{n\pi}{a}x\right)\Psi(x,0)\,dx$

from $\Psi(x,0) \to \Psi(x,t)$

$p = m\frac{d}{dt}x$

$\langle p \rangle = m\frac{d}{dt}\langle x \rangle$

$\langle x \rangle = \int_{-\infty}^{\infty} x\rho(x,t)\,dx = \int_{-a}^{a} x|\Psi|^2 dx$

expectation-value

$\langle x^2 \rangle = \int x^2|\Psi|^2 dx$

$\langle p^2 \rangle = \int p^2|\Psi|^2 dx$

$\langle \hat{O}[\Psi] \rangle = \int \Psi^* \hat{O}[\Psi]\,dx$

operator e-v

$\langle H \rangle = \sum_{n=1}^{\infty} |C_n|^2 E_n \approx E_1$

$\hat{P}[\ ]\Psi = p\Psi$ - eigenvalue

Quantum Bohr

$\vec{L} = \sqrt{l(l+1)}\,\hbar$ $\cos\theta = \frac{L_z}{|L|}$

$L_z = m\hbar$ n, l, m

$E_n = \frac{-13.6\,eV}{n^2}$ $n[0, \ldots]$

Degeneracy $= n^2$ $l[0, n-1]$

$m[-l, l]$

$\Psi_{nlm}(r,\theta,\phi) = R_{nl}(r)\Theta_{lm}(\theta)\Phi_m(\phi)$

$|\vec{S}| = \sqrt{s(s+1)}\,\hbar$ $s = +\frac{1}{2}, -\frac{1}{2}$

$S_z = \pm\frac{\hbar}{2}$ $2l$ or $(2l+1)$ etc

$E_{eff} = (Z_{eff})^2 \cdot \frac{13.6\,eV}{n^2}$

3D TISE (sphere) $-\frac{\hbar^2}{2m}\nabla^2\Psi + U(r)\Psi = E\Psi$

$\Delta E = -\vec{\mu}_B \cdot \vec{B}_{ext}$ $\vec{\mu}_B = \frac{e\hbar}{2m_e}$

Zeeman Effect (spin influenced by magnetic field $= \Delta E$)

Bohr Magneton (Magnetic Moment)

Infinite Square Well:

$\Psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$ $n = 1, 2, 3, \ldots$

$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

Lyman → 1 Balmer → 2 Paschen → 3

$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

cartesian polar

$z = a + bi = re^{i\theta}$

$|z|$ $\tan^{-1}\left(\frac{b}{a}\right)$

$z = r\cos\theta + ir\sin\theta = re^{i\theta}$

$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

Quantum Oscillator

$E_n = (n + \frac{1}{2})\hbar\omega$

$U(x) = \frac{1}{2}m\omega^2 x^2$

$\omega = \sqrt{\frac{k}{m}}$

Light Amplification Stimulated Emission of (E/M) Radiation