$T_K = T_C + 273$

Physics 120 Equation Sheet
(Thermal Physics)

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State Equations

$R = 8.31 \frac{J}{mol \cdot K}$

$PV = nRT$ ← Ideal Gas Law
$V = NK_BT$

· Van der Waals

$\left(P + \frac{a}{v^2}\right)(v - b) = RT$ ← v: "molar" volume

· Beattie-Bridgeman

$P = \frac{RT(1-\varepsilon)}{v^2} - (v+B) - \frac{A}{v^2}$

Laws of Thermodynamics

·1. $\Delta E = \Delta Q \pm \Delta W$ $\quad dS = \frac{1}{T}dQ$
$\Delta E = T\Delta S \pm P\Delta V$ $\quad dW = PdV$

·2. $dS = \frac{1}{T}dQ$ (entropy increases)
$\Delta S > 0$

·3. $T = 0,\ S = 0$

UHAG (Energy Functions)

$dU = TdS - PdV$
$dH = TdS + VdP$ $\quad H \equiv U + PV$
$dA = -SdT - PdV$ $\quad A \equiv U - TS$
$dG = -SdT + VdP$ $\quad G \equiv H - TS$

Maxwell Relations

$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$

$\left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$

$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

Statistics + Prob

$\Omega(N,n) = \binom{N}{n} = \frac{N!}{n!(N-n)!}$
multiplicity of macrostate
N: # of particles
n: # of possible states

Einstein solid:

$\Omega(N,q) = \binom{q+N-1}{q} = \frac{(q+N-1)!}{q!(N-1)!}$

$\Omega(N,q) = \left(\frac{eq}{N}\right)^N$ (for $q \gg N$) (high-temp limit)

Sterling's Approx

$\ln(N!) = N\ln(N) - N + \frac{1}{2}\ln(2\pi N)$
$N! = N^N e^{-N}\sqrt{2\pi N}$ → $N \gg 1$

$\ln\left(1+\frac{a}{b}\right) = \frac{a}{b}$ for $a \ll b$

w/Entropy $S \equiv K_B \ln(\Omega_{tot})$
$\Omega_{tot} = \Omega_A \cdot \Omega_B$

Sackur-Tetrode (Entropy of ideal gas)

$S = Nk_B\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$

Kinetic Theory of Gasses

Gas = particles moving, colliding w/walls!

$V_{rms} = \sqrt{\frac{3k_BT}{m}}$ ← Microscopic V_{rms} ↕ Macroscopic T

$\langle KE \rangle = \frac{1}{2}mV_{rms}^2 = \frac{3}{2}K_BT$ ← $K_B = \frac{R}{N_A}$ $\quad N_A = 6.02 \times 10^{23} \frac{1}{mol}$

$K_B = 1.38 \times 10^{-23} \frac{J}{K}$

$\boxed{\langle E \rangle = N \cdot f \cdot \frac{1}{2}K_BT}$ ← Equipartition Theorem!

$\left(C_V = \frac{f}{2}K_B\right)$

$\boxed{f = \text{\# of degrees freedom}}$

3x translational $\frac{1}{2}mv^2$ + 2x rotational $\frac{1}{2}I\omega^2$ + 2x vibrational $\frac{1}{2}Kx^2$

f = 3 → 5 → 7

Heat Capacity

$C_V = \left(\frac{dE}{dT}\right)_V \quad C_P = \left(\frac{dE}{dT}\right)_P$ ← const. V or P

C = heat cap. = cm (specific heat)

↳ un-calculus-ify: $\Delta E = C\Delta T$

$\boxed{\gamma = \frac{C_P}{C_V}}$ $\quad \boxed{C_P = C_V + nR}$

Adiabatic $PV^\gamma = \text{const.}$

C_V: Trans $\frac{3}{2}$, Rot $\frac{5}{2}$, Vib $\frac{7}{2}$ vs. T
f = 3, +2, +2
3, 5, 7

$C_V = \frac{f}{2}(N)K_B$ $\quad N = nN_A$

$C_P = C_V + (N)K_B$

Compressibility

$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ ← Bulk modulus $B = \frac{1}{K_T}$

$K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S$ $\quad \frac{K_T}{K_S} = \frac{C_P}{C_V}$

Chemical Potential

$\mu_{chem} \equiv -T\left(\frac{\partial S}{\partial n}\right)_{U,V}$ ← $n = \frac{N}{N_A}$

Statistical Mechanics

$Z = \sum_{ASN} e^{-\frac{E}{K_BT}} = \sum g_i e^{-\frac{E_i}{K_BT}}$

$P(E_i) = \frac{g_i e^{-E_i/K_BT}}{Z}$ ← gibbs Boltzmann factor

Quantum Distributions: $\bar{n} = \frac{1}{e^{\frac{(E-\mu)}{KT}} + a}$

a = 0 (Maxwell-Boltzmann)
a = +1 (Fermi-Dirac)
a = -1 (Bose-Einstein)

$Z_{grand} = \sum_{ASN} e^{-(E_i - \mu N)/KT}$

$\varepsilon_F = KT_F$

$V_Q = \left(\frac{h}{\sqrt{2\pi m KT}}\right)^3$

$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$ (ε_F of ideal gas)