# Physics 112 Equation Sheet

[illegible]
Fall 2022

↳ Electrodynamics (E&M 2)

## Current / Current densities

$\vec{I} = \lambda \vec{v}$ — secret equation: $\left(\lambda = \frac{Q}{L} = \frac{dq}{d\ell}\right)$

$\boxed{\vec{J} = \rho \vec{v}} = \frac{d\vec{I}}{da}$  $\boxed{\vec{K} = \sigma \vec{v}} = \frac{I}{\ell}$ (perp)

$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ — Continuity eq. (Charge conserved)

## Magnetization

$\vec{M}$ = magnetization = $\frac{\text{Mag. dipole moment}}{\text{Volume}}$

$\vec{J}_b = \vec{\nabla} \times \vec{M}$  $\vec{K}_b = \vec{M} \times \hat{n}$

$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ ← Aux field (free current)

$\vec{J}_f = \vec{\nabla} \times \vec{H}$  $I_{free_{enc}} = \oint \vec{H} \cdot d\vec{\ell}$ — find H with symmetry

$M = \chi_m H$  $\vec{B} = \mu \vec{H} = \mu_0(\vec{H} + \vec{M})$

$\chi_m$: Magnetic susceptibility (to be magnetized)  $\mu = \mu_0(1 + \chi_m)$

$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$

## Emf

$\mathcal{E} = -\frac{d\Phi_B}{dt} = \oint E \cdot d\ell$  $\Phi_B = \int \vec{B} \cdot d\vec{a}$

$\vec{F}_B = q\vec{v} \times \vec{B}$  $F_{B_{wire}} = \int I\, d\vec{\ell} \times \vec{B}$

$\vec{E}_f = -\frac{\partial \vec{A}}{\partial t}$ ← Faraday Field

## Biot-Savart Law

$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \hat{r}}{r^2}} = \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \hat{r}}{r^2} da = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \hat{r}}{r^2} d\tau$

Main (I)

## Ampere's Law

$\oint \vec{B}_{wire} \cdot d\vec{\ell} = \mu_0 I_{enc} = \mu_0 \int \vec{J} \cdot d\vec{a} = \mu_0 K \ell$ (perp. len, parallel)

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ — weird form (full Maxwell's eqns form)

## Vector Potential

$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}}{r} d\tau = \frac{\mu_0}{4\pi} \int \frac{\vec{K}}{r} da = \frac{\mu_0}{4\pi} \int \frac{\vec{I}}{r} d\ell$

$\vec{B} = \vec{\nabla} \times \vec{A} \leftrightarrow \vec{A} = \frac{1}{4\pi} \int \frac{\vec{B} \times \hat{r}}{r^2} d\tau$

$\oint A \cdot d\ell = \oint (\nabla \times B) \cdot d\ell = \oint B \cdot d\vec{a}$  $\nabla \cdot A = 0$

$\left.\frac{\partial A_{in}}{\partial z}\right|_{z=a} - \left.\frac{\partial A_{out}}{\partial z}\right|_{z=a} = -\mu_0 K$

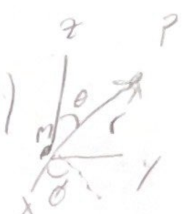
$\left.A_{in}\right|_{z=a} = \left.A_{out}\right|_{z=a}$

## Magnetic Dipole

$m = I \int da = Ia$ — Mag. dipole moment.

$\vec{A}_{dipole} = \frac{\mu_0}{4\pi} \frac{(\vec{m} \times \hat{r})}{r^2}$

$\vec{B}_{dipole} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\, \hat{r} + \sin\theta\, \hat{\theta})$




## Laws of Reflection / Refraction

$\theta_I = \theta_R$ (law of reflection)  $n_1 \sin\theta_I = n_2 \sin\theta_T$ (Snell's Law)

$\tilde{\vec{E}}(\vec{r},t) = \tilde{\vec{E}}_0 e^{i(kr - \omega t)} \hat{n}$, $\tilde{\vec{B}}(\vec{r},t) = \frac{1}{v}(\hat{k} \times \tilde{\vec{E}})$

$v = \frac{1}{\sqrt{\epsilon \mu}} = \frac{c}{n}$  $n \equiv \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}}$  $\tilde{\vec{B}}_0 = \frac{1}{v}(\hat{k} \times \tilde{\vec{E}}_0)$

$k = \frac{2\pi}{\lambda}$  $\omega = 2\pi f$  $v = f\lambda \, (=c)$

$c^2 = \frac{1}{\mu_0 \epsilon_0}$

Coeff. of Reflection/Transmission

$\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$  $\alpha \equiv \frac{\cos\theta_T}{\cos\theta_I}$  $R \equiv \frac{I_R}{I_I}$  $T \equiv \frac{I_T}{I_I}$  $I_I = \frac{1}{2}\epsilon_1 v_1 E_{0_I}^2 \cos\theta_I$

## Gauge Transformations

$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda \qquad \vec{V}' = \vec{V} - \frac{\partial\lambda}{\partial t}$$

Both Potential sets yield same fields.

Coulomb Gauge if $\vec{\nabla}\cdot\vec{A} = 0$

Lorenz Gauge if $\vec{\nabla}\cdot\vec{A} = -\mu_0\varepsilon_0\frac{\partial V}{\partial t}$

## Scalar Potential

(electrostatics, integral form)

Coulombs Law

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho}{r^2}\hat{r}\,d\tau = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma}{r^2}\hat{r}\,dA = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda}{r^2}\hat{r}\,dl$$

Gauss's Law

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{1}{r}\rho\,d\tau = \frac{1}{4\pi\varepsilon_0}\int\frac{1}{r}\sigma\,dA = \frac{1}{4\pi\varepsilon_0}\int\frac{1}{r}\lambda\,dl$$

$$\oint E\cdot dl = \frac{Q_{enc}}{\varepsilon_0}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$