Math 14 Equation Sheet

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dt^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}}$$

Polar coordinates

$dA = r\,dr\,d\theta$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \cos^{-1}\left(\frac{x}{r}\right) \\ \theta = \sin^{-1}\left(\frac{y}{r}\right) \end{cases}$$

Symmetry

x-axis: $f(r,\theta) = f(r,-\theta)$

y-axis: $f(r,\theta) = f(r,\pi-\theta)$

origin: $f(r,\theta) = f(-r,\theta)$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

Trig Identities

$$\cos^2\theta = \frac{\cos(2\theta)+1}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

Arc length $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \dots}\; dt$

$$\vec{v}(x) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\hat{T} = \frac{\vec{v}(x)}{|\vec{v}(x)|}$$ ← tangential unit vector

Spherical coordinates

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$r = \rho\sin\phi$ (shortcut)

$\rho^2 = x^2+y^2+z^2$

$dV = \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

Work Integrals?

HW 7 → $\int\sqrt{1-x^2}\,dx = \frac{1}{2}\sin^{-1}(x) - \frac{1}{2}x\sqrt{1-x^2}$

Line Integrals

$ds = |\vec{v}(t)|\,dt$

$$\int_C f(x,y,z)\,ds = \int_a^b f(x,y,z)\,|\vec{v}(t)|\,dt$$

vector form

$$\int_C \vec{F}\cdot \hat{T}\,ds = \int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}\cdot\frac{d\vec{r}}{dt}\,dt = \int_C M\,dx + \int_C N\,dy + \int_C P\,dz$$

$$\int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$ – Fundamental Theorem of Line Integrals

Component Test ($\vec{F}$ is conservative) $M\hat{i}, N\hat{j}, P\hat{k}$

$$\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z} \text{ and } \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \text{ and } \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

→ If is conservative:

$$\vec{F} = \nabla f \to \langle M, N, P\rangle = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right\rangle$$

$\oint \vec{F}\cdot d\vec{r} = 0$ – loop property

Coordinate Transformations

$$\iint_R f(x,y)\,dx\,dy = \iint_G f(g(u,v), h(u,v))\,|J(u,v)|\,du\,dv$$ (abs)

Jacobian Determinant

$$J(u,v) = \frac{\partial(x,y)}{\partial(u,v)} = \det\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}$$

remember $|J(u,v)|$

Green's Theorem

ccw circulation (flow) version

$$\oint_C \vec{F}\cdot\hat{T}\,ds = \oint_C M(x,y)\,dx + N(x,y)\,dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dA$$ ($dx\,dy$)

outward flux version

$$\oint_C \vec{F}\cdot\hat{n}\,ds = \oint_C M\,dy - N\,dx = \iint_R \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\,dA$$

$$SA = \iint_D |\vec{r}_u\times\vec{r}_v|\,du\,dv = \iint d\sigma, \quad d\sigma = |\vec{r}_u\times\vec{r}_v|\,du\,dv$$

$$A = \oint_C \tfrac{1}{2}x\,dy - \oint_C \tfrac{1}{2}y\,dx$$

$\iint_S \vec{F}\cdot\hat{n}\,d\sigma$ – surface integral

$$\hat{n} = \frac{\vec{r}_u\times\vec{r}_v}{|\vec{r}_u\times\vec{r}_v|}$$

Double Integrals

Fubini's Theorem

$dA = dy\,dx$

$dA = r\,dr\,d\theta$ ↳ Polar

$$\int_{y=c}^{y=d}\int_{x=a}^{x=b} f(x,y)\,dx\,dy = \int_{x=a}^{x=b}\int_{y=c}^{y=d} f(x,y)\,dy\,dx$$

$$\iint_D (1)\,dA = \text{Area of } D$$ ($z=1$)

$$AVG = \frac{1}{b-a}\int_a^b f(x)\,dx \quad 2D$$

$$AVG = \frac{1}{\text{Area}}\iint_D f(x,y)\,dA \quad 3D$$

$$Avg = \frac{1}{vol}\iiint_E f(x,y,z)\,dV$$

Triple Integrals

$$M = \iiint_D \delta(x,y,z)\,dV \qquad V = \iiint_E (1)\,dV$$

Moments

$$M_{yz} = \iiint_D x\,\delta(x,y,z)\,dV$$

$$M_{xz} = \iiint_D y\,\delta(x,y,z)\,dV$$

$$M_{xy} = \iiint_D z\,\delta(x,y,z)\,dV$$

$CM = (\bar{x}, \bar{y}, \bar{z})$

$\bar{x} = \frac{M_{yz}}{M}, \dots$

$2D \to CM = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$

Stokes Theorem

Curl - vector

if $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$

$$\text{curl } \vec{F} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\hat{i} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x}\right)\hat{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\hat{k} = \vec{\nabla} \times \vec{F}$$

del $\vec{\nabla} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$

operator

Divergence

$$\text{div } \vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} = \vec{\nabla} \cdot \vec{F}$$

div curl $\vec{F} = 0$

always

Stokes: line integral $\leftrightarrow$ surface integral

circulation

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\vec{\nabla} \times \vec{F}) \cdot \hat{n} \, d\sigma$$

$$\iint_S \vec{F} \cdot \hat{n} \, d\sigma = \iiint_D \vec{\nabla} \cdot \vec{F} \, dV$$ Divergence Theorem

Divergence Theorem

$$\iint_S \vec{F} \cdot \hat{n} \, d\sigma = \iiint_D \vec{\nabla} \cdot \vec{F} \, dV$$