Phys 111

Legendre Polynomials:

$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = (3x^2 - 1)/2$
$P_3(x) = (5x^3 - 3x)/2$
$P_4(x) = (35x^4 - 30x^2 + 3)/8$
$P_5(x) = (63x^5 - 70x^3 + 15x)/8$

Identities and Properties:

$$P_l(-x) = (-1)^l P_l(x)$$

$$(2l+1)P_l(x) = \frac{d}{dx}(P_{l+1}(x) - P_{l-1}(x))$$

$$\int_0^1 P_l(x)dx = \frac{P_l'(0)}{l(l+1)}$$

$P_l(1) = 1$ (normalization condition)

$P_l(x) = \frac{1}{2^l l!}(\frac{d}{dx})^l(x^2-1)^l$ (Rodrigues formula)

Legendre's Equation
↳ $\frac{d}{dx}(1-x^2)\frac{d}{dx}P_\ell(x) + \ell(\ell+1)P_\ell(x) = 0$

FUNDAMENTAL CONSTANTS

ϵ_0	=	$8.85 \times 10^{-12}\,C^2/Nm^2$	(permittivity of free space)
μ_0	=	$4\pi \times 10^{-7}\,N/A^2$	(permeability of free space)
c	=	$3.00 \times 10^8\,m/s$	(speed of light)
e	=	$1.60 \times 10^{-19}\,C$	(charge of the electron)
m	=	$9.11 \times 10^{-31}\,kg$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi} \\ \hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi} \\ \hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}(\sqrt{x^2+y^2}/z) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z} \\ \hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z} \\ \hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{x} = \cos\phi\,\hat{s} - \sin\phi\,\hat{\phi} \\ \hat{y} = \sin\phi\,\hat{s} + \cos\phi\,\hat{\phi} \\ \hat{z} = \hat{z} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{s} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y} \\ \hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y} \\ \hat{z} = \hat{z} \end{cases}$$

$$\Psi(x,t) = \sum_{n=0}^{\infty} c_n \sqrt{\tfrac{2}{a}} sin(\tfrac{n\pi}{a}x) e^{-iE_n t/\hbar} \qquad \Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x) \qquad c_n = \sqrt{\tfrac{2}{a}} \int_0^a sin(\tfrac{n\pi}{a}x)\Psi(x,0)dx$$

$$V(x) = \tfrac{1}{2}m\omega^2 x^2 \qquad E_n = (n+\tfrac{1}{2})\hbar\omega \quad |\vec{L}| = \sqrt{l(l+1)}\hbar \quad L_z = m\hbar \quad |\vec{S}| = \sqrt{s(s+1)}\hbar \quad S_z = m_s\hbar$$

<u>Mathematical Formulas:</u>

$$\int_0^\infty \tfrac{x^3}{e^x-1}dx = \pi^4/15, \quad \int_0^\infty \tfrac{x^2}{e^x-1}dx \cong 2.40, \quad \int_0^\infty x^n e^{-bx}dx = \tfrac{n!}{b^{n+1}} \quad \int_0^\infty x^n e^{-x/b}dx = n!\, b^{n+1}$$

$$\int_0^\infty x^{2n} e^{-x^2/b^2}dx = \sqrt{\pi}\tfrac{2n!}{n!}(\tfrac{b}{2})^{2n+1} \qquad \tfrac{3}{2}! = \tfrac{3}{4}\sqrt{\pi}, \quad \tfrac{1}{2}! = \tfrac{1}{2}\sqrt{\pi}, \quad 0! = 1, \quad e^{i\theta} = cos\theta + i\ sin\theta$$

$$\int x sin^2(bx)dx = -\tfrac{(2bx(sin(2bx)-ax)+cos(2bx))}{(8b^2)}, \quad \int x^2 sin^2(bx)dx = \tfrac{(4b^3x^3+(3-6b^2x^2)sin(2bx)-6bxcos(2bx))}{(24b^3)}$$

$$\int sin^2(bx)dx = \tfrac{x}{4} - \tfrac{1}{4b}sin(2bx) \qquad \int x sin(bx)dx = \tfrac{1}{b^2}sin(bx) - \tfrac{x}{b}cos(bx) \qquad sin(a\pm b) = sin(a)cos(b) \pm cos(a)sin(b)$$

$$\sum_{n=0}^{\infty} ne^{-n\alpha} = -\tfrac{d}{d\alpha}\sum_{n=0}^{\infty} e^{-n\alpha}, \quad \tfrac{1}{1-x} = 1 + x + x^2 + x^3...., \quad e^x \cong 1 + x + x/2! + x^3/3!...$$

<u>Constants:</u>

$c = 3\times10^8$ m/s	1 eV $= 1.6\times10^{-19}$ J
$h = 6.626\times10^{-34}$ J·s $= 4.14\times10^{-15}$ eV·s	$\hbar = h/2\pi = 1.055\times10^{-34}$ J·s $= 6.58\times10^{-16}$ eV·s
$hc = 1240$ eV· nm	$\frac{h}{mc} = 0.0243$ Å
$k_B = 1.38\times10^{-23}$ J/K $= 8.617\times10^{-5}$ eV/K	$\sigma = 5.67\times10^{-8}$ W/m²K⁴
$m_{e^-} = 9.11\times10^{-31}$ kg $\cong .511$ MeV/c²	$e^- = -1.6\times10^{-19}$ C
$R = 1.097\times10^7$ m⁻¹	$a_0 = 0.529$ Å $= 0.0529$ nm
$k = 9.0\times10^9$ N·m²/C²	G $= 6.67\times10^{-11}$ m³ kg⁻¹ s⁻²
$\epsilon_0 = 8.85\times10^{-12}$ F/m	$\mu_0 = 4\pi\times10^{-7}$ H/m

Group → Period ↓	1	2		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca		21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr		39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
			*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb		
			**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No		